Higher-dimensional charged black holes solutions with a nonlinear electrodynamics source

Mokhtar Hassaïne*

Instituto de Matemática y Física, Universidad de Talca, Casilla 747, Talca, Chile, and
Laboratoire de Mathématiques et de Physique Théorique,
Université de Tours, Parc de Grandmont, Tours, France.

Cristián Martínez†

Centro de Estudios Científicos (CECS), Casilla 1469, Valdivia, Chile
and Centro de Ingeniería de la Innovación del CECS (CIN), Valdivia, Chile.

We obtain electrically charged black hole solutions of the Einstein equations in arbitrary dimensions with a nonlinear electrodynamics source. The matter source is deriving from a Lagrangian given by an arbitrary power of the Maxwell invariant. The form of the general solution suggests a natural partition for the different ranges of this power. For a particular range, we exhibit a class of solutions whose behavior resemble to the standard Reissner-Nordström black holes. There also exists a range for which the black hole solutions approach asymptotically the Minkowski spacetime slower than the Schwarzschild spacetime. We have also found a family of not asymptotically flat black hole solutions with an asymptotic behavior growing slower than the Schwarzschild (anti) de Sitter spacetime. In odd dimensions, there exists a critical value of the exponent for which the metric involves a logarithmic dependence. This critical value corresponds to the transition between the standard behavior and the solution decaying to Minkowski slower than the Schwarzschild spacetime.

I. INTRODUCTION

One of the most interesting black hole solutions with a matter source is the Kerr-Newman solution. The Kerr-Newman solution describes a massive and rotating charged black hole, which is solution of the Einstein-Maxwell equations in four dimensions. At the vanishing angular momentum limit, the Kerr-Newman solution reduces to the Reissner-Nordström solution. In four dimensions, the Maxwell theory is conformally invariant and hence, the derivations of the Reissner-Nordström or the Kerr-Newman solutions are considerably simplified since in this case the scalar curvature vanishes. In higher dimensions, where the Maxwell theory is not conformally invariant, the derivation of the corresponding Reissner-Nordström solution has been done long time ago [1]. However, finding the equivalent of the Kerr-Newman in higher dimensions is still an open problem. Recently, we have taken seriously the lack of conformal symmetry of the Maxwell action for dimensions $d > 4$ and proposed the following generalization of the Maxwell action in arbitrary dimensions

$$I_{\mathcal{M}} = -\alpha \int d^d x \sqrt{-g} \, (F_{\mu\nu} F^{\mu\nu})^q \,, \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the Maxwell tensor, and α is a constant [2]. The interest of considering such action lies in the fact that in arbitrary dimension d, the action $I_{\mathcal{M}}$ can enjoy the conformal invariance provided the exponent to be chosen as $q = d/4$. For this particular choice of the exponent, we have derived black hole solutions electrically charged with a purely radial electric field of the conformal nonlinear electrodynamics for dimensions of multiples of four. The restriction on the dimension is due to the electromagnetic field Ansatz which imposes the exponent to be given only by multiples of four in order to deal with real solutions [2]. A way of escaping from this restriction is to consider the absolute value of the Maxwell Lagrangian as it has been done in three dimensions with the conformal exponent [3]. Note that there exists another conformally invariant extension of the Maxwell action in higher dimensions for which the Maxwell field is replaced by a $d/2-$form with d even [4]. The black hole solutions of this theory were discussed in [5].

In the present paper, we relax the conformal condition and instead, we consider the Einstein equations in arbitrary dimension with the nonlinear electrodynamics source (1) without restricting *à priori* the exponent q. The option

of considering nonlinear electrodynamics models as sources of the Einstein equation is motivated by the fact that nonlinear electrodynamics models provide excellent laboratories for the construction of black hole solutions with interesting properties, for instance, regular black holes [6]. Also, in the last years, a number of papers have been dedicated to find and analyze different solutions in gravitational theories with nonlinear electrodynamics sources, including, in some cases, additional matters fields [7].

The plan of the paper is organized as follows. In Section II, after presenting the field equations, we determine the sign of the coupling constant α in terms of the exponent q in order to satisfy the energy conditions. We derive the most general static and spherically symmetric solution with a purely radial electric field in Sec. III. The general solution contains two integration constants and it is possible to find event horizons, which also depend on q and the spacetime dimension. Moreover, as it is expected, the two integration constants of the general solution can be related with the mass and electric charge and the higher-dimensional generalization of the Reissner-Nordström solution is obtained as the exponent q is set to 1. As it is shown along of this section, the general solution suggests a natural partition for the different ranges of the exponent q.

First of all, there is an excluded region corresponding to the values $q \in (0, 1/2)$ for which the scalar curvature of the solutions diverges at infinity. For $q \in (1/2, (d-1)/2) \cap \tilde{\mathbb{Q}}$, where the set $\tilde{\mathbb{Q}}$ denotes the rational number with odd denominator (5), the solution has a behavior similar to the standard Reissner-Nordström solution. By similar we mean that asymptotically the charge contribution in the metric goes to zero faster than the mass contribution. At the opposite and surprisingly enough, black hole solutions which go asymptotically to the Minkowski spacetime slower than the Schwarzschild spacetime are also exhibited for $q > (d-1)/2$ and for $q < -1/(d-4)$ with $q \in \tilde{\mathbb{Q}}$. In four dimensions, these solutions only exist for $q > 3/2$ with $q \in \tilde{\mathbb{Q}}$. For the remaining values of the exponent, i. e. $q \in (-1/(d-4), 0) \cap \tilde{\mathbb{Q}}$, the black hole solutions are not asymptotically flat and, their asymptotic behavior is shown to grow slower that the Schwarzschild (anti) de Sitter spacetime. In four dimensions, these not asymptotically flat black hole solutions are exhibited for all non positive exponents belonging to the set $\tilde{\mathbb{Q}}$.

The different black hole solutions are classified, depending on their asymptotic behaviors, among these four different classes. The transitions between these different classes of solutions are operated at the critical values $q_{c_1} = (d-1)/2$ and $q_{c_2} = -1/(d-4)$, and it is clear that only in odd dimensions these critical values belong to the set $\tilde{\mathbb{Q}}$. Thus, an analysis of the solutions at the critical exponents is done in odd dimensions. At the first critical exponents q_{c_1} which corresponds to the transition between the standard behavior and the solution decaying to Minkowski spacetime slower than the Schwarzschild spacetime, it is shown that the metric involves a logarithmic dependence and the expression of the electric potential is proportional to $\ln r$ independently of the odd dimensions. At the other critical value q_{c_2}, the transition is smooth and corresponds to the region where the metric function asymptotically goes to a constant (not necessarily equal to 1) proportional to the black hole charge. After this detailed study of all the cases, the Sec. IV is concerned with the comments and the possible extensions of the present work.

II. FIELD EQUATIONS

We consider the following action in $d > 2$ dimensions

$$I[g_{\mu\nu}, A_\mu] = \int d^d x \sqrt{-g} \left[\frac{R}{2\kappa} - \alpha \left(F_{\mu\nu} F^{\mu\nu} \right)^q \right], \tag{2}$$

where q is a rational number that will be fixed later, R is the scalar curvature, and $\kappa > 0$ is the gravitational constant. The field equations obtained by varying the metric and the gauge field A_μ read respectively

$$G_{\mu\nu} = 4\kappa\alpha \left[q F_{\mu\rho} F_\nu{}^\rho \, F^{q-1} - \frac{1}{4} g_{\mu\nu} \, F^q \right], \tag{3a}$$

$$\frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} \, F^{\mu\nu} \, F^{q-1} \right) = 0, \tag{3b}$$

where F is the Maxwell invariant $F = F_{\alpha\beta} F^{\alpha\beta}$.

We are looking for a static and spherically symmetric spacetime geometry whose line element is given by

$$ds^2 = -N^2(r) f^2(r) \, dt^2 + \frac{dr^2}{f^2(r)} + r^2 \, d\Omega_{d-2}^2, \tag{4}$$

where $d\Omega_{d-2}^2$ is the line element of the $(d-2)-$dimensional sphere. In addition, we only consider a purely radial electric Ansatz for the electromagnetic field which means that the only non-vanishing component of the Maxwell

tensor is given by F_{tr}. As a direct consequence, the Maxwell invariant $F = -2(F_{tr})^2$ is negative and hence, the exponent q can only be an integer or a rational number with odd denominator. Hence, in order to deal with real solutions, the exponent q is restricted to be an element of the following set [1]

$$\tilde{\mathbb{Q}} = \left\{ \frac{n}{2p+1}, \quad (n,p) \in \mathbb{Z} \times \mathbb{Z} \right\}. \tag{5}$$

Taking the trace of the Einstein equations, the scalar curvature is expressed in terms of the Maxwell invariant F as

$$R = 2\kappa\alpha\frac{(4q-d)}{(2-d)}F^q. \tag{6}$$

Using this expression for the scalar curvature, the Einstein equations can be written as

$$G^t_t = -\frac{((Nf)'f)'}{N} - (d-2)\frac{(Nf)'f}{rN} - \kappa\alpha\frac{(4q-d)}{(2-d)}F^q = \kappa\alpha(2q-1)F^q \tag{7a}$$

$$G^r_r = -\frac{((Nf)'f)'}{N} - (d-2)\frac{f'f}{r} - \kappa\alpha\frac{(4q-d)}{(2-d)}F^q = \kappa\alpha(2q-1)F^q \tag{7b}$$

$$G^{\theta_i}_{\theta_i} = -\frac{(Nf)'f}{rN} - \frac{f'f}{r} + \frac{d-3}{r^2}(1-f^2) - \kappa\alpha\frac{(4q-d)}{(2-d)}F^q = -\kappa\alpha F^q \tag{7c}$$

where θ_i, with $i = 1, \cdots, (d-2)$, are the angular coordinates and the prime denotes derivative with respect to the radial coordinate r. Subtracting Eqs. (7a) and (7b) we obtain that

$$-(d-2)\frac{N'f^2}{rN} = 0, \tag{8}$$

hence $N(r)$ is a constant, which can be set to 1 without loss of generality.

Before studying in details the field equations, we first specify the sign of the coupling constant α in term of the exponent q in order to ensure a physical interpretation of our future solutions. In fact, the sign of the coupling constant α in the action (2) can be chosen such that the energy density, i.e. the $T^{\hat{0}\hat{0}}$ component of the energy-momentum tensor in the ortonormal frame, is positive

$$T^{\hat{0}\hat{0}} = -\kappa\alpha(2q-1)F^q > 0.$$

This condition selects two branches depending on the range of the exponent q,

$$\begin{cases} \mathrm{sgn}(\alpha) = -(-1)^q & \text{for} \quad q > 1/2, \\ \mathrm{sgn}(\alpha) = (-1)^q & \text{for} \quad q < 1/2, \end{cases} \tag{9}$$

while the case $q = 1/2$ is excluded by the condition (5).

III. ELECTRICALLY CHARGED BLACK HOLE SOLUTIONS

Let us now derive the most general solution of the static and spherically symmetric Einstein equations (3) with a purely radial electric field. As said previously, the substraction of the equations (7a) and (7b) implies that the metric function N can be set to 1. As a direct consequence, the generalized Maxwell equation (3b) reads

$$\partial_r \left(r^{d-2} F^{rt} (F_{rt})^{2q-2} \right) = 0, \tag{10}$$

which can easily be solved yielding to $F_{tr} \propto r^{\frac{2-d}{2q-1}}$. Hence, the Einstein equations become differential equations of the only metric function f and their integration yields to the following solution

$$F_{tr} = \frac{C}{r^{\frac{d-2}{2q-1}}}, \tag{11a}$$

$$f^2(r) = 1 - \frac{A}{r^{d-3}} - \frac{2\kappa\alpha(-1)^q C^{2q} 2^q (2q-1)^2}{(d-2)(d-2q-1)r^{\frac{2(qd-4q+1)}{2q-1}}}, \tag{11b}$$

$$N(r) = 1, \tag{11c}$$

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[1] Note that $\mathbb{Z} \subset \tilde{\mathbb{Q}}$.

where A and C are two integration constants proportional to the mass and the electric charge. Various comments can be made concerning the structure of this solution. Firstly, as it can be seen from the metric function (11b), the odd dimensions $d = 2n + 1$ with $q = n$ requires a special analysis. In fact, as it will be shown below this critical value corresponds to the transition between the standard behavior and the solution decaying to Minkowski slower than the Schwarzschild spacetime. The solutions derived here reduce to the higher-dimensional generalizations of the Reissner-Nordström solution [1] for $q = 1$ as it should be since this limiting case is not singular. In the conformal situation, i. e. $q = d/4$, the solutions (11) exist only for dimensions of multiples of four because of the restriction (5), and our previous results [2] are recovered. It is also interesting to note that the Schwarzschild (anti) de Sitter solution can be obtained from the expression (11b) by putting $q = 0$. This result is not surprising since at the level of the action (2), putting $q = 0$ is equivalent of considering the Einstein action with a cosmological constant Λ given in term of the coupling constant α as $\Lambda = \kappa\alpha$.

We are interested in finding solutions with event horizons. These horizons should be hide curvature singularities, and hence solutions having singularities at infinity will be ruled out and only curvature singularities surrounded by an event horizon are allowed. For the solution (11), the scalar curvature has a single singularity at the origin $r = 0$ if $q \in \tilde{\mathbb{Q}}$ and $q > 1/2$ or $q \leq 0$, while the range $0 < q < 1/2$ is excluded since the scalar curvature diverges at the infinity. The analysis of the Kretschmann invariant, $K = R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho}$, leads to the same conditions on q in order to satisfy the regularity condition at infinity. Moreover, it is easy to show that the curvature regularity condition as $r \to \infty$, in addition to the positive energy condition, ensure that the weak energy condition is satisfied.

The existence of the horizons for the solutions (11b) must be done carefully because of the presence of various parameters as the constants appearing in the action, the dimension d and the exponent q, as well as the integration constants A and C. Hence, for clarity we first rewrite the metric function (11b) schematically as

$$f^2(r) = 1 - \frac{A}{r^{d-3}} + \frac{B}{r^\beta} \tag{12}$$

where we have defined

$$B = -\frac{2\kappa\alpha(-1)^q C^{2q} 2^q (2q-1)^2}{(d-2)(d-2q-1)}, \qquad \beta = \frac{2(qd - 4q + 1)}{2q - 1}. \tag{13}$$

The form of the metric solution suggests two natural ranges concerning the exponent β, namely whether if $1/r^\beta$ goes faster or not to zero than the Schwarzschild potential $1/r^{d-3}$, i.e. $\beta > d - 3$ or $0 < \beta < d - 3$, respectively. The remaining possibilities are $\beta = d - 3$ which corresponds to the critical exponent $q = (d-1)/2$ and $\beta \leq 0$ for which the metric is not asymptotically flat. All these different possibilities are now analyzed in detail.

A. Case $\beta > d - 3$

The case $\beta > d - 3$ is similar to the standard Reissner-Nordström case in the sense that for large r, the metric behaves like the Schwarzschild metric. The condition $\beta > d - 3$ imposes the exponent q to be in the following range $q \in \tilde{\mathbb{Q}} \cap (1/2, (d-1)/2)$ which in turn implies that the electric field (11a) vanishes at infinity. On the other hand, the positivity of the energy density (9) imposes the constant B to be positive. In this case, in order to have real roots for the metric function $f^2(r)$, the constant A must be positive and the constant B must be chosen in the following range

$$0 < B < (d-3)\left(\frac{A}{\beta}\right)^{\frac{\beta}{d-3}}(\beta + 3 - d)^{\frac{\beta+3-d}{d-3}}. \tag{14}$$

Under these conditions, we have two roots localized at $r_- \in (0, b)$ and $r_+ \in (b, \infty)$ where

$$b = \left(\frac{A}{\beta}(\beta + 3 - d)\right)^{\frac{1}{d-3}}.$$

An extreme black hole can also be obtained if A is positive and the constant B given by

$$B = (d-3)\left(\frac{A}{\beta}\right)^{\frac{\beta}{d-3}}(\beta + 3 - d)^{\frac{\beta+3-d}{d-3}}.$$

B.   Case $0 < \beta < d - 3$

The case $0 < \beta < d - 3$ is compatible with exponents q given by

$$q \in \tilde{\mathbb{Q}} \cap (\frac{d-1}{2}, \infty) \qquad \text{or} \qquad q \in \tilde{\mathbb{Q}} \cap (-\infty, \frac{-1}{d-4})$$

This situation is surprisingly enough since the Schwarzschild potential $1/r^{d-3}$ goes faster to zero than the potential proportional to the electric charge. The energy condition implies that the constant B is always negative, and hence if the constant A is positive, there is a single root

$$r \in]r_0, \infty[, \qquad r_0 = \left(\frac{-B(d-3-\beta)}{d-3}\right)^{1/\beta}$$

On the other hand if the constant $A < 0$, there are two roots if $A > A_0$ where

$$A_0 = -\frac{\beta}{d-3-\beta}\left(\frac{-B(d-3-\beta)}{d-3}\right)^{(d-3)/\beta} \tag{15}$$

and a double root if $A = A_0$.

The emergence of black hole solutions for $0 < \beta < d - 3$ is interesting by itself since the solutions approach asymptotically the Minkowski spacetime slower than the Schwarzschild spacetime. Such asymptotic behavior has been observed in the case of massive scalar fields minimally coupled to anti de Sitter gravity with a potential [8].

Another amusing fact is concerned with the odd dimensions given by $d = 7 + 4p$ where $p \in \mathbb{N}$ and $q = -(2+p)/(1+2p)$ for which the metric solution becomes

$$f^2(r) = 1 + \frac{\kappa\,\alpha\,(-1)^{q+1}\,C^{2q}\,2^{q-1}}{(p+1)\,(2p+1)\,r^{2p+2}} - \frac{A}{r^{2(2p+2)}}.$$

It is clear from this expression that, up to the physical interpretation of the integration constants, this metric function corresponds to Reissner-Nordström metric function in odd dimension $\tilde{d} = 2p + 5 = (d+3)/2$. This analogy is clearly not respected by the electric field since the Maxwell electric field in the nonlinear case is given by $F_{tr} = C\,r^{2p+1}$ and does not correspond to the Reissner-Nordström solution in dimension $\tilde{d} = 2p + 5$.

C.   Case $\beta = d - 3$

The limiting case $\beta = d-3$ corresponds to the change of phase between the solution which resembles to the standard Reissner-Nordström solution and the one for which the asymptotic decaying to Minkowski spacetime is slower than the Schwarzschild spacetime. This case is compatible with the values of the exponent q given by $q = (d-1)/2$, and because of the restriction (5), the exponent can only be a positive integer $q \geq 1$. This implies that for the critical exponent $q = (d-1)/2$, we are only concerned with odd dimensions $d = 2q + 1$. This value is critical in the sense that the constant B of the solution (13) is singular at this value. In this case, the integration of the Einstein equations for a static and spherically symmetric spacetime in presence of a purely radial electric field yields

$$F_{tr} = \frac{C}{r}, \tag{16a}$$

$$f^2(r) = 1 - \frac{A}{r^{2q-2}} + \kappa\alpha(-1)^q 2^{q+1} C^{2q} \frac{\ln r}{r^{2q-2}}, \tag{16b}$$

where A and C are two constants of integration. For a constant $A > 1$, the existence of horizons for the logarithmic solution (16) is always ensured. In the remaining case $A \leq 1$, the metric function (16b) can have roots provided one fixes properly the constant C. Some comments can be added concerning this particular solution. The three-dimensional case corresponds to the standard Maxwell case, i.e. $q = 1$, and the expression (16) reduces to the three-dimensional Einstein-Maxwell solution [9]. The emerging of a logarithmic function in the metric is similar to the case studied in [10], where the slow asymptotic fall-off produced by these logarithmic branches appear in the context of a self-interacting scalar field whose mass saturates the Breitenlohner-Freedman bound, minimally coupled to Einstein gravity with a negative cosmological constant in $d > 2$ dimensions. It is also interesting to remark that for any odd dimensions $d > 3$, the solution approaches to Minkowski space in the asymptotic region, but it is not asymptotically flat due to the presence of a logarithmic term. Also, it is remarkable that independently of the value

of the exponent $q > 1$, the expression of the electric field does not depend on the dimension. This means that in any odd dimensions $d = 2n + 1$ and with $q = n$, the electric field behaves like the Maxwell field in three dimensions. A same analogy has been observed in dimensions $d = 4n$ with $q = n$, where the expression of the electric field is the same than the four-dimensional Coulomb field [2]. The hierarchical character of the electric field of the solutions, as the exponent q takes integer values, can be generalized in the following sense. In arbitrary dimension $\tilde{d}$, the electric field of the Reissner-Nordström solution goes like $F_{tr} \propto 1/r^{\tilde{d}-2}$ [1]. From the generic solution (11a), it is clear that such behavior can be obtained in dimensions $d \geq \tilde{d}$ such that

$$d = (\tilde{d} - 2)(2q - 1) + 2,$$

with the exponent $q \in \mathbb{N}$.

D. Case $\beta < 0$

It remains to study the case for which the metric solution (11) is not asymptotically flat, i.e. $\beta < 0$. First of all, the condition $\beta < 0$ is consistent only with $q \in (-1/(d-4), 1/2)$ and, since we are not considering the solutions having singularities at the infinity, this interval is restricted to be $q \in (-1/(d-4), 0)$. On the other hand, the positivity of the energy density (9) implies that the constant B in (13) is negative. In this situation, for a constant $A > 0$, the metric function will have two roots provided that the constant A is bounded as follows

$$A < A_0, \qquad A_0 = \frac{-\beta}{d - 3 - \beta} \left(\frac{-(d-3)}{B(d-3-\beta)} \right)^{(3-d)/\beta}$$

while a double root is obtained for $A = A_0$, however, this does not lead to an extreme black hole. In the case for which $A < 0$, there is a single root localized in the following interval $]r_0, \infty[$ where

$$r_0 = \left(\frac{-(d-3)}{B(d-3-\beta)} \right)^{-1/\beta}.$$

Another interesting fact is that the metric function behaves asymptotically like $f(r)^2 \sim r^{-\beta}$ with $0 < -\beta < 2$ which in turn implies that the metric function goes to infinity slower than the Schwarzschild (anti) de Sitter spacetime. The only option for the metric function to go faster to infinity than the Schwarzschild (anti) de Sitter geometry is for $q \in (0, 1/2)$ which is precisely the forbidden region where the solution has a naked singularity.

IV. DISCUSSION

In this paper, we have seen the influence of considering arbitrary power of the Maxwell Lagrangian as a source of the Einstein equations for a static and spherical symmetric spacetime geometry. We have obtained the most general black hole solutions of the Einstein equations for static and spherically symmetric spacetime with a purely radial electric field. The general solutions depend on the dimension, the exponent and reduce to the Reissner-Nordström solutions as the exponent $q = 1$. We have exhibited some interesting properties of these black hole solutions. The range of the exponent q has been divided in four parts depending on the asymptotic behavior of the solution. For example, we have derived black hole solutions that go asymptotically to the Minkowski spacetime faster or slower than the Schwarzschild solution. We have also found a range for which the black hole solutions are not asymptotically flat generalizing the standard Schwarzschild (anti) de Sitter solution without requiring the introduction of a cosmological constant. There also exists a critical value of the exponent only in odd dimensions for which the metric solution involves a logarithmic dependence and the behavior of the electric field does not depend on the dimension and is given by the standard three-dimensional Maxwell field.

It will be interesting to realize a similar analysis in the case of a static and axisymmetric Ansatz for the metric and to look for rotating charged black hole solution. There are essentially two interesting options to explore in this perspective. The first one is to consider the four-dimensional problem with an arbitrary power of the Maxwell Lagrangian in order to obtain a metric generalizing the Kerr-Newman geometry, and also to see the influence of the exponent on the asymptotic behavior of the metric. Mathematically, this work is highly non trivial because the field equations are complicated and, in contrast with the Einstein-Maxwell case, the matter source in the nonlinear case is not longer conformally invariant. The other option is to consider the same problem in higher dimensions but with a conformal nonlinear source, i. e. $q = d/4$. In the static and spherical case and for a purely electric field, we have

already solved this problem [2]. In the same spirit, we can also look for other kind of solutions as for example the charged Taub-NUT solutions in even dimensions [11].

In the metric solution (11), there appear two constants of integration that are proportional to the mass and the electric charge. It will be interesting to identify precisely the mass and the electric of the black hole solutions. In the case of solutions which are asymptotically flat, the use of the Hamiltonian action may provide a simple manner of identifying the black hole mass and charge which is not necessarily the case for the solutions with $\beta < 0$.

Another interesting aspect to explore will be the stability of the solutions derived here. In this context, the authors of Ref. [12] have derived conditions on a class of the electromagnetic Lagrangians to ensure the linear stability of black hole configurations.

Finally, it is also be desirable to study the geometric properties, the causal structures as well as the thermodynamics properties of the black hole solutions derived here. In particular, these questions may be of interest in the case of the solutions derived for which the spacetimes are not asymptotically flat.

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